space becomes increasingly stratified and, for sufficiently high Rayleigh numbers (order of 10^9), the flow down the cold wall does not penetrate this region at all, but instead separates from the wall toward the baffle tip, thus forming a weak recirculation in the lower-baffle/cold-wall region.

Nusselt Number Results

Figure 3 compares the hot- and cold-wall Nusselt numbers for different aperture locations. Also included in the figure is Bajorek and Lloyd's[11] experimentally determined correlation for an air-filled enclosure with h_ℓ/H equal to 0.25 and aperture ratio of 0.5. As expected, the perfectly conducting predictions with $h_\ell/H = 0.25$ agree reasonably well with the correlation, while the predictions with the adiabatic end walls are substantially higher. The lowest heat transfer occurs for $h_\ell/H = 0.5$ and Nu increases with decreasing h_ℓ/H. Similar behavior is noted along the cold wall, except that when the end walls are conducting the average Nusselt number appears to be relatively insensitive to the value of h_ℓ/H. To explain some of these trends, it should be noted that, as h_ℓ/H is increased, thermal stratification in the lower-baffle/cold-wall region becomes stronger, resulting in lower heat transfer between the fluid stream and the lower portion of the cold wall. The local Nusselt number along the hot wall also decreases with increasing h_ℓ/H. At a higher value of h_ℓ/H the flow detaches earlier from the cold wall; therefore, the flow up the hot wall is correspondingly less cold, leading to a decrease in Nusselt number.

Conclusion

A numerical study has been made to determine the influence of the aperture location in an enclosure with centrally located upper and lower baffles and dimensionless aperture opening of 0.5. The aperture location is found to have a significant influence on the heat-transfer, velocity, and temperature profiles along the cold wall, but a weaker influence on the corresponding quantities along the hot wall.

References

[1]Probert, S.D. and Ward, J., "Improvements in Thermal Resistance of Vertical, Air-Filled, Enclosed Cavities," *Proceedings of the Fifth International Heat Transfer Conference,* Japanese Society of Mechanical Engineering, Tokyo, NC3.9, Sept. 1974, pp. 124-138.

[2]Janikowski, H.E., Ward, J., and Probert, S.D., "Free Convection in Vertical, Air Filled Rectangular Cavities Fitted With Baffles," *Proceedings of the 6th International Heat Transfer Conference*, Hemisphere Publishing, Washington, DC, 1978, pp. 257-262.

[3]Nansteel, M.W. and Greif, R., "Natural Convection in Undivided and Partially Divided Rectangular Enclosures," *Transactions of ASME, Journal of Heat Transfer,* Vol. 103, Nov. 1981, pp. 623-629.

[4]Nanstell, M.W. and Greif, R., "An Investigation of Natural Convection in Enclosures with Two- and Three-Dimensional Partitions," *International Journal of Heat and Mass Transfer*, Vol. 27, No. 4, 1984, pp. 561-571.

[5]Lin, N.L. and Bejan, A., "Natural Convection in a Partially Divided Enclosure," *Internal Journal of Heat and Mass Transfer,* Vol. 26, No. 12, 1983, pp. 1867-1878.

[6]Winters, K.H., "The Effect of Conducting Divisions on the Natural Convection of Air in a Rectangular Cavity with Heated Side Walls, *American Society of Mechanical Engineering*, Paper 82-HT-69.

[7]Duxbury, D., "An Interferometric Study of Natural Convection in Enclosed Plane Air Layers with Complete and Partial Central Vertical Division" Ph.D. Thesis, University of Salford, England, 1979.

[8]Winters, K.H., "Laminar Natural Convection in a Partially-Divided Rectangular Cavity at a High Rayleigh Number," *Journal of Fluid Mechanics* (submitted for publication).

[9]Kelkar, S. and Patankar, S.V., "Numerical Prediction of Natural Convection in Partitioned Enclosures," *American Society of Mechanical Engineers*, New York, Pub. HTD-Vol. 63, 1986, pp. 63-71.

[10]Chang, L.C., "Finite Difference Analysis of Radiation-Convection Interactions in Two-Dimensional Enclosures," Ph.D. Thesis, Dept. of Aerospace and Mechanical Engineering, University of Notre Dame, IN.

[11]Bajorek, S.M. and Lloyd, J.R., "Experimental Investigation of Natural Convection in Partitioned Enclosures," *Transactions of ASME, Journal of Heat Transfer*, Vol. 104, Aug. 1982, pp. 527-532.

[12]Zimmerman, E.B. and Acharya, S., "Free Convection Heat Transfer in a Partially Divided Vertical Enclosure with Conducting End Walls," *International Journal of Heat and Mass Transfer,* Vol. 30, No. 2, 1987, pp. 319-331.

[13]Jetli, R., Acharya, S., and Zimmerman, E.B., "The Influence of Baffle Location Natural Convection in a Partially Divided Enclosure," *Numerical Heat Transfer,* Vol. 10, No. 5, 1986, pp. 521-536.

[14]Elder, J.W., "Laminar Free Convection in a Vertical Slot," *Journal of Fluid Mechanics,* Vol. 23, Pt. 1, 1965, pp. 77-98.

[15]Landis, F. and Yanowitz, H., "Transient Natural Convection in a Narrow Vertical Cell," *Proceedings of the Third International Heat Transfer Conference,* American Institute of Chemical Engineers, New York, 1966.

[16]Linthorst, S.J.M., Schinkel, W.M.M., and Hoogendorn, C.J., "The Stratification in Natural Convection in Vertical Enclosures," *Natural Convection in Enclosures,* edited by K.E. Torrance and I. Catton, American Society of Mechanical Engineers, New York, Pub. HTD-Vol. 8, 1980, pp. 31-38.

[17]Zimmerman, E.B. and Acharya, S., "Natural Convection in a Vertical Square Enclosure with Perfectly Conducting End Walls," *Proceedings of the ASME Solar Energy Conference*, American Society of Mechanical Engineers, New York, 1986, pp. 57-65.

[18]Patankar, S.V., *Numerical Heat Transfer and Fluid Flow,* McGraw-Hill, New York, 1980.

Energy Transfer in an Elliptic Annulus with Uniform Heat Generation

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Nomenclature

C,c	= dimensional and dimensionless constant temperature gradients, Eq. (9)
E,e	= dimensional and dimensionless excess temperatures, Eq. (9)
E_{in},e_{in}	= dimensional and dimensionless excess temperatures of the inner pipe
k	= thermal conductivity, $W/(m^2-K)/m$
L	= radius of outer periphery, m
m	= ellipticity of the annulus
P,P	= dimensional and dimensionless pressure, Eq. (2)
Pe	= Peclet number, $Re_L Pr$
Pr	= Prandtl number, ν/α
q	= characteristic heat flux, Eq. (9)
Q	= mass flow rate through the annulus, kg/s
Re_L	= Reynolds number, WL/ν
T	= temperature, $[CZ+E(X,Y)]$, K
T_0,T_i	= outer and inner wall temperatures, K
$X,x=X/L$	= dimensional and dimensionless transverse coordinate

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$Y, y = Y/L$ = dimensional and dimensionless transverse coordinate
$Z, z = Z/L$ = dimensional and dimensionless longitudinal coordinate
α = thermal diffusivity, m^2/s
μ = dynamic viscosity, kg/(m − s)
ν = kinematic viscosity, m^2/s
ξ = dimensionless elliptic coordinate,

$$x = \xi[1 + (m^2/\xi^2)] \cos\eta$$

η = dimensionless elliptic coordinate,

$$x = \xi[1 + (m^2/\xi^2)] \cos\eta$$

ω = core size or dimensionless inner radius of the annulus defined as the ratio of the inner radius to the outer radius

Introduction

THE problem studied here follows Ref. 1 and is the complete form of Ref. 2. A special case of this study, where ellipticity m equals zero, is given in Ref. 3. The velocity and temperature distributions with fully developed profiles are presented in closed forms. The heat flux for the inner and outer walls are obtained explicitly. The numerical results are given in graphical form. In Ref. 4, results for an elliptic tube with constant wall temperature are presented. A recent literature survey[5] showed that no analytical or experimental work was done in this area. The subject matter covered here has importance in a large variety of traditional engineering disciplines, i.e., nuclear engineering and compact heat exchanger design.

Viscous Flow in Annular Elliptic Pipes

The velocity distribution for an incompressible, constant-property fluid in laminar flow inside a confocal elliptic annulus in regions away from the inlet where the velocity profile is considered fully developed is given by[1]

$$W = Re_L (W_0 - W_2 \cos 2\eta) \tag{1}$$

where

$$\frac{dP}{dZ} = \frac{\mu\nu}{L^3}\frac{dp}{dz}, \qquad Re_L = -\frac{1}{4}\frac{dp}{dz} \tag{2}$$

$$W_0 = (1-\xi^2)\left(1-\frac{m^4}{\xi^2}\right) - \left[\left(1-\frac{m^4}{\omega^2}\right)(1-\omega^2)\Big/\log\omega\right]\log\xi \tag{3}$$

$$W_2 = \frac{2m^2}{1+\omega^2}(1-\xi^2)\left(1-\frac{\omega^2}{\xi^2}\right) \tag{4}$$

The rate of mass flow through the annulus of the elliptic pipes is

$$Q = 2\pi\mu L Re_L I_{00} \tag{5}$$

where

$$I_{00} = \frac{1}{4}(1-\omega^4)\left(1+\frac{m^8}{\omega^4}\right) - 2m^4\frac{(1-\omega^2)}{(1+\omega^2)} - \frac{1}{4}(1-\omega^2)\left(1-\frac{m^4}{\omega^2}\right)C_1 \tag{6}$$

$$C_1 = -\left[\left(1-\frac{m^4}{\omega^2}\right)(1-\omega^2)\right]\Big/\log\omega \tag{7}$$

Temperature Distribution

The temperature distribution in a flow between confocal elliptic boundaries each maintained under constant wall heat flux conditions must have the following functional form[1] of $T = CZ + E(X,Y)$ where $E = 0$ on the outer periphery and $E = E_{in}$ on the inner periphery, which requires each wall temperature to be constant at any cross section. Note that for noncircular cross sections, peripherally uniform temperature distributions do not correspond to uniform heat flux distributions around the peripheries.[1]

Including an arbitrary heat generation term W_{GEN} and neglecting viscous dissipation, the energy equation to be satisfied is

$$V\cdot \text{grad}\,T = \alpha\left(\nabla^2 T + \frac{Q_{GEN}}{k}\right) \tag{8}$$

The variables involved in the analysis are nondimensionalized as

$$Q_{GEN} = \frac{q}{L}q_{GEN}, \qquad C = \frac{q}{k}c, \qquad E = \frac{Lq}{k}e \tag{9}$$

Introducing Eqs. (7) and (9) into Eq. (8), one obtains the dimensionless energy equation in elliptic coordinates as

$$\left(\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial\eta^2}\right)e = -q_{GEN}(X_2 - 2m^2\cos 2\eta) + G(E_0 - E_2\cos 2\eta + E_4\cos 4\eta) \tag{10}$$

where

$$X_2 = \xi^2 + \frac{m^4}{\xi^2}, \qquad G = cPe, \qquad E_0 = x_2 w_0 + m^2 w_2$$

$$E_2 = 2m^2 w_0 + x_2 w_2, \qquad E_4 = m^2 w_2 \tag{11}$$

The general solution to the energy equation is given by Ref. 2 as

$$e = e_{in} g_0 + q_{GEN}(f_0 + f_2\cos 2\eta) + G(e_0 + e_2\cos 2\eta + e_4\cos 4\eta) \tag{12}$$

where

$$g_0 = \frac{\log\xi}{\log\omega}; \qquad f_0 = \frac{1}{4}w_0; \qquad f_2 = -\frac{1}{4}w_2 \tag{13}$$

$$\begin{aligned} e_0 = &-\frac{1}{4}w_0(1+m^4) + \frac{1}{4}w_2 m^2 \\ &+ \frac{1}{4}C_1\log\xi\left(1-\frac{\omega^2}{\xi^2}\right)\left(\xi^2-\frac{m^4}{\omega^2}\right) \\ &+ \frac{1}{16}\left[(1-\xi^4)\left(1-\frac{m^8}{\xi^4}\right) + C_1\log\xi(1+\omega^2)\left(1+\frac{m^4}{\omega^2}\right)\right] \\ &+ \frac{1}{4}C_1(1-\xi^2)\left(1+\frac{m^4}{\xi^2}\right) \\ &+ \frac{1}{4}C_1^2\log\xi\left[\left(1+\frac{m^4}{\omega^2}\right)\Big/\left(1-\frac{m^4}{\omega^2}\right)\right] \end{aligned} \tag{14}$$

$$e_2 = -\frac{1}{6}\frac{m^2}{(1+\omega^2)}\left[(1-\xi^4)\left(1-\frac{m^4\omega^2}{\xi^4}\right)+C_1(1+\omega^2)\log\xi\right]$$
$$+\frac{2}{3}\frac{m^2}{(1+\omega^2)}\left[C_1(1+\omega^2)\log\xi+(1-\xi^2)\left(1-\frac{m^4\omega^2}{\xi^2}\right)\right.$$
$$\left.-\frac{\omega^4}{(1+\omega^2)}\left(1-\frac{m^4}{\omega^4}\right)\left(\xi^2-\frac{1}{\xi^2}\right)\right] \tag{15}$$

$$e_4 = -\frac{1}{6}\frac{m^4}{(1+\omega^2)}(1-\xi^2)\left(1-\frac{\omega^2}{\xi^2}\right)$$
$$+\frac{1}{24}\frac{m^4}{(1+\omega^4)}(1-\xi^4)\left(1-\frac{\omega^4}{\xi^4}\right) \tag{16}$$

Heat Fluxes Through the Walls

The element of heat flux dU measured in the positive direction of ξ through an elemental area of $\xi=$constant elliptic surface is

$$dU = -Lq\xi\frac{\partial e}{\partial \xi}d\eta \tag{17}$$

The energy transfer rates U_i and U_o per unit length of inner and outer pipes, respectively, which are taken to be positive when heat flows into the fluid, are expressed as

$$U_i = -Lq\omega\int_o^{2\pi}\left(\frac{\partial e}{\partial \xi}\right)_{\xi=\omega}d\eta \tag{18}$$

$$U_o = Lq\int_o^{2\pi}\left(\frac{\partial e}{\partial \xi}\right)_{\xi=1}d\eta \tag{19}$$

Substituting from Eq. (12),

$$U_i = -2\pi Lq[\beta G+\omega q_{GEN}f_o'(\omega)\log\omega$$
$$+\omega Ge_o'(\omega)\log\omega]/\log\omega \tag{20}$$

$$U_o = 2\pi Lq[\beta G+q_{GEN}f_o'(1)\log\omega+Ge_o'(1)\log\omega]/\log\omega \tag{21}$$

where $f_o'(\omega)$, $f_o'(1)$, $e_o'(\omega)$, and $e_o'(1)$ are the derivatives of f_o and e_o with respect to ξ evaluated at $\xi=\omega$ and $\xi=1$ and are given in the Appendix. Also,

$$\beta = \frac{E_{in}}{LcPe} \tag{22}$$

Two special values of β are given as follows:
1) For insulated outer wall, $U_o=0$:

$$\beta=\beta_i=-\frac{1}{G}[q_{GEN}f_o'(1)\log\omega+Ge_o'(1)\log\omega] \tag{23}$$

2) For insulated inner wall, $U_i=0$:

$$\beta=\beta_o=-\frac{1}{G}[\omega q_{GEN}f_o'(\omega)\log\omega+\omega Ge_o'(\omega)\log\omega] \tag{24}$$

The differences of Eqs. (23) and (24) can be written as

$$\beta_i-\beta_o=-\frac{q_{GEN}\log\omega}{G}[f_o'(1)-\omega f_o'(\omega)]$$
$$-\log\omega[e_o'(1)-\omega e_o'(\omega)] \tag{25}$$

Using Eqs. (A5) and (A6), the preceding equation reduces to

$$\beta_i-\beta_o=\frac{q_{GEN}\log\omega}{2G}(1-\omega^2)\left(1+\frac{m^4}{\omega^2}\right)-I_{00}\log\omega \tag{26}$$

Let the ratio of the heat gain rate from the outer wall to that from both walls per unit length of pipe be denoted by λ. Then

$$\lambda=(\beta-\beta_i)/(\beta_o-\beta_i) \tag{27}$$

which depends only on the dimensionless inner-wall temperature. The value of λ for the special case of equal wall temperatures can be defined in terms of the inner wall temperature T_i and the outer wall temperature T_o and

$$E_{in}=(T_i-T_o) \text{ at } \beta=0 \tag{28}$$

Therefore, the value of λ from Eq. (30) becomes

$$\lambda_o=\beta_i/(\beta_i-\beta_o) \tag{29}$$

Introducing the ratio $\bar{\mu}=\lambda/\lambda_o$, β can be expressed as

$$\beta=-(1-\bar{\mu})\left[\frac{1}{G}q_{GEN}f_o'(1)\log\omega+e_o'(1)\log\omega\right] \tag{30}$$

Upon the substitution of this value of β into Eqs. (20) and (21) for the inner and outer heat fluxes, respectively, the results become

$$U_i=2\pi Lq\left\{-\frac{1}{2}q_{GEN}(1-\omega^2)\left(1+\frac{m^4}{\omega^2}\right)+I_{00}G\right.$$
$$\left.-\bar{\mu}[q_{GEN}f_o'(1)+Ge_o'(1)]\right\} \tag{31}$$

$$U_o=2\pi Lq\{\bar{\mu}[q_{GEN}f_o'(1)+Ge_o'(1)]\} \tag{32}$$

For the special cases considered, the values of λ, β, and $\bar{\mu}$ are as follows:
1) For insulated outer wall:

$$U_o=0,\quad \lambda=0,\quad \beta=\beta_i,\quad \bar{\mu}=0$$

2) For equal wall temperatures:

$$T_o=T_i,\quad \lambda=\lambda_o,\quad \beta=0,\quad \bar{\mu}=1$$

3) For insulated inner wall:

$$U_i=0,\quad \lambda=1,\quad \beta=\beta_o,\quad \bar{\mu}=\bar{\mu}_o=1/\lambda_o$$

Numerical Results and Conclusions

Energy transfer in steady laminar convection in the annulus of two confocal elliptical pipes is analyzed, and the results are presented in analytical closed form. Equations (31) and (32) show that U_i and U_o depend on five independent parameters, namely the magnitude of heat generation q_{GEN}, the ellipticity of the pipe—the core size ω, the Reynolds number, the Prandtl number, and the wall temperature gradient c.

The heat gains are plotted vs core size of the pipe, ω, for constant Peclet number and uniform heat generation and with one ellipticity, $m=0.5$. This is presented in Fig. 1. The case of a circular annulus, $m=0$, is given in Ref. 3. The graphical results are only for one value of the heat generation density that is characteristic of nuclear phenomenon. This nondimensional value is 0.719 and was calculated using

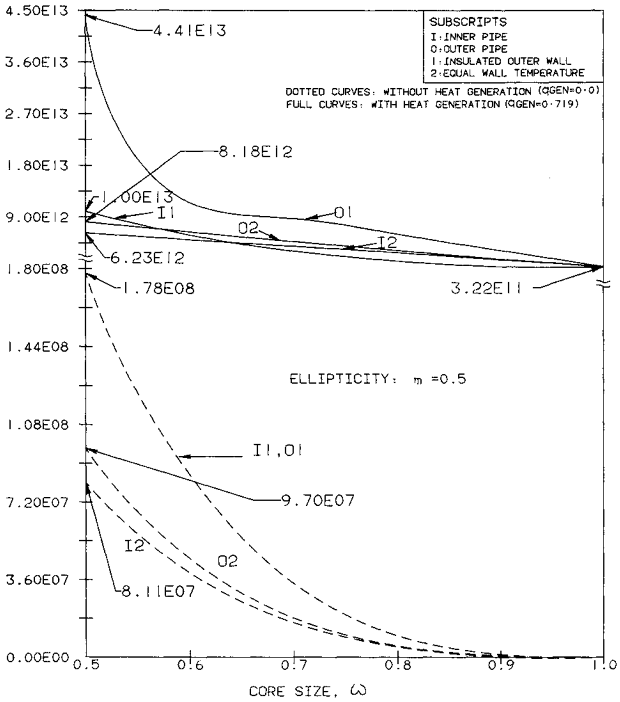

Fig. 1 Energy transfer vs core size for ellipticity of 0.5.

data given in Ref. 6. It is indeed easy to evaluate Eqs. (31) and (32) numerically for any other generation density. The intent is to show the effect of heat generation in the annulus on energy transfer through the walls of a confocal elliptical annulus.

Appendix

$$f_o'(1) = \frac{1}{4}\,[C_1 - 2(1-m^4)] \tag{A1}$$

$$f_o'(\omega) = \frac{1}{4}\left[\frac{C_1}{\omega} - 2\left(\omega - \frac{m^4}{\omega^3}\right)\right] \tag{A2}$$

$$\begin{aligned} e_o'(1) = {} & \frac{1}{4}(1-m^8) - \frac{1-\omega^2}{1+\omega^2}\, m^4 - \frac{1}{4}\, C_1(1+m^4) \\ & - \frac{3}{16}\, C_1(1+\omega^2)\left(1+\frac{m^4}{\omega^2}\right) \\ & + \frac{1}{4}\, C_1^2\left(1+\frac{m^4}{\omega^2}\right)\Big/ 1-\frac{m^4}{\omega^4}\Big) \end{aligned} \tag{A3}$$

where

$$\begin{aligned} \omega e_o'(\omega) = {} & -\frac{1}{4}\,(1+m^4)\left[G - 2\left(\omega^2 - \frac{m^4}{\omega^2}\right)\right] \\ & + \frac{1-\omega^2}{1+\omega^2}\, m^4 + \frac{1}{2}\, C_1\left(\omega^2 - \frac{m^4}{\omega^2}\right)\log\omega \\ & - \frac{1}{4}\left[\left(\omega^2 - \frac{m^8}{\omega^4}\right) + \frac{1}{16}\, C_1(1+\omega^2)\left(1+\frac{m^4}{\omega^2}\right)\right] \\ & - \frac{1}{2}\, C_1\left(\omega^2 + \frac{m^4}{\omega^2}\right) + \frac{1}{4}\, C_1^2\left(1+\frac{m^4}{\omega^2}\right)\Big/\left(1-\frac{m^4}{\omega^2}\right) \end{aligned} \tag{A4}$$

$$f_o'(1) - \omega f_o'(\omega) = -\frac{1}{2}\,(1-\omega^2)\left(1+\frac{m^4}{\omega^2}\right) \tag{A5}$$

$$e_o'(1) - \omega e_o'(\omega) = I_{00} \tag{A6}$$

References

[1]Topakoglu, H. D. and Arnas, O. A., "Convective Heat Transfer for Steady Laminar Flow Between Two Confocal Elliptic Pipes with Longitudinal Uniform Wall Temperature Gradient," *International Journal of Heat Mass Transfer*, Vol. 17, 1974, pp. 1487–1498.

[2]Arnas, O. A. and Ebadian, M. A., "Energy Gain by Conduction for a Confocal Elliptic Annulus with Uniform Wall Temperature Gradient and Heat Generation," *Proceedings, Ninth Canadian Congress of Applied Mechanics*, 1983, pp. 603–604.

[3]Arnas, O. A. and Ebadian, M. A., "Energy Gain by Conduction for Concentric Circular Annuli with Uniform Wall Temperature Gradient and Heat Generation," *Transactions of the 7th International Conference on Structural Mechanics in Reactor Technology*, Vol. B, 1983, pp. 143–150.

[4]Ebadian, M. A., Topakoglu, H. C., and Arnas, O. A., "On the Convective Heat Transfer in a Tube of Elliptic Cross Section Maintained Under Constant Wall Temperature," *Journal of Heat Transfer*, Vol. 108, 1986, pp. 33–39.

[5]Eckert, E. R. G., Goldstein, R. J., Pfender, E., Ibele, W. E., Patankar, S. V., Ramsey, J. W., Simon, T. W., and Deckor, N. A., "Heat Transfer—A Review of the 1982 Literature," *International Journal of Heat Mass Transfer*, Vol. 26, 1983, pp. 1733–1770.

[6]Dudeerstadt, J. J. and Hamilton, L. J., *Nuclear Reactor Analysis*, 1st Edition, Appendix H, John Wiley, New York, pp. 634–635.

Computational Aspects of Heat Transfer in Structures via Transfinite-Element Formulation

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Introduction

THE computational aspects of transient thermal analysis of structures has been approached by several researchers[1,2] using finite elements; the most common being the step-by-step time-marching schemes, modal superposition techniques, etc. The present Note describes a viable computational approach with emphasis on a generalized transfinite-element methodology[3] for applicability for the computational aspects of heat transfer in structures. Highlights and characteristic features of the approach are described via general formulations and applications to sample one- and two-dimensional problems. Comparative numerical test cases therein validate the fundamental capabilities via the proposed concepts.

Transform-Methods-Based Finite-Element Methodology

Briefly summarized, the transform-methods-based finite-element approach combines the modeling versatility of contemporary finite elements in conjunction with transform methods and classical Galerkin schemes. In this Note, the formulations are applied for transient thermal analysis; however, the general concepts are applicable to other transient and/or interdisciplinary problems. First, the region under consideration is idealized as a finite number of discrete elements. Therein, numerical

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